Fusion Cells

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Based on the resonant ion confinement concept, for a Deuteron cloud in a Penning-Malmberg trap with a specially configured rotating wall, the possibility to build a new kind of fusion reactor is analysed. It is proved that, for some trap configurations, nuclear fusion reactions will take place in the center of the trap vessel. In that case, Lawson criterion for an efficient fusion reactor is satisfied. Moreover, the reactor will have a compact design and, since it does not require a large facility for its implementation, we call this device as a Fusion Cell.

PhySH: Laboratory plasma, Penning traps, Fusion reactors.

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I. INTRODUCTION

For an efficient fusion reactor the pressure p of the confined plasma times the energy confinement time τ_E should be greater than a given amount

$$p\tau_E > L. \tag{1}$$

A criterium due to Lawson (Ref. [1]). For Deuterium - Deuterium reactions this value is of the order of one hundred atm $\times$ s. As a matter of fact, in the case of the main sequence stars it is gravity what provides the confinement conditions to effectively satisfy such criterium. Thus, given that Deuterium is an stable and abundant Hydrogen isotope on Earth, being able to artificially confine these nuclei in a controlled and sustainable way, also satisfying the fusion conditions, and, thereby, taking advantage of the energy of the stars, is the "raison d'être" of nuclear fusion research. Notwithstanding with that, instead of the instance of pure Deuterium, a much more energetically favored reaction possibility comes from the case of the fusion of Deuterium and Tritium and, in that case, the magnetic confinement of neutral plasmas has been thought as the more promising concept to build a fusion reactor. In Tokamaks, the magnetic field confines the particles and energy long enough for ignition to occur and it is expected that such a reactor will generate enough energy to achieve $Q > 10$, which will probably be shown through the ITER experiment in the near future,(see Ref. [2], also Ref. [3]). Those expectations overcome the challenges inherent to the Tokamak concept, namely the possibility of suffering disruptions, the existence of edge localised modes (ELMs) and the intrinsic pulsed working, on top of the possible impurity accumulation, (as from Ref. [4]). Turbulence will be also playing a major role degrading the Tokamak plasma confinement. These problems make it necessary to explore other possible magnetic confinement fusion concepts like the stellarator one (see e.g. Ref. [5]), characterised by enabling a continuous working and by the absence of large ELMs and disruptions, since there plasmas are almost currentless. Stellarator confinement is one generation after the tokamak one, due to the fact that they have to reduce their neoclassical transport by optimization, as well as to solve the confinement of fast particles and to demonstrate power exhaust by a suitable divertor concept. Therefore, it is still necessary to overcome difficulties in both concepts to make it economically feasible for the production of electricity by fusion. Other topic to discuss in the future would be the size of the reactors, since all the scalings show an improvement of confinement with the size of the device (see e.g. Ref. [6] and references therein).

A possible alternative and expectedly much less costly method to achieve nuclear fusion is described in this article. The research on this concept can be considered as a risk mitigation action in case that the main stream lines do not reach the expected results, and opens the possibility to build small size fusion reactors. It is based on the concept of Resonant Ion Confinement (see Ref. [7], that can happen inside a cylindrical Penning-Malmberg ion trap). In this device ions are confined by applying an electric potential difference,V_0, together with a magnetic axial field $\mathbf{B}$. The ion trap, additionally, has a quadrupole electric field of a frequency, ω, such that it allows the Deuterium nuclei to rotate also with that same angular velocity. In this system, a fraction of ions close to the centre of the device performs trajectories that will bring them close enough to produce fusion reactions by tunnelling. The confined ion system takes the shape of a highly flattened spheroid. Although these characteristics are common to any Penning-Malmberg system, there is a very precise set of values for the trap parameters that determine the intensity of the quadrupole field for which

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resonant, coalescing paths, arise among all the Deuterium ion trajectories, and, in this way, nuclear fusion reactions will take place. Thus, the precise dependence of this quadrupole electric field as a function of the electric field potential, V_0, and of the magnetic field intensity, $\mathbf{B}$, as well as the radius of the trap cylinder, R_0, will determine the resonance conditions required for the system to produce a finite number of fusion reactions. Finally, the pressure and confinement time necessary for ignition in the center of the ionic trap will be reached only for special configurations of the confinement system parameters. This summarizes the features of a new candidate for compact nuclear fusion reactor: a Fusion Cell.

The article is organised as follows. First we summarize the dynamic properties of a charged plasma of Deuterons confined in a resonant configured Penning-Malmberg trap (the more technically involved features of that can be found extensively in the Appendix). Next we review the power balance of the fusion reactions in this context. Thereafter, the thermodynamic constrains that the self-sustained reactor shall satisfy is described and, following this, the actually attainable electric power, for a realistic implementation, is estimated. We end the article with some conclusions about the technical possibilities of the fusion cell concept.

II. RESONANT ION CONFINEMENT.

Let us consider N Deuteron nuclei of mass m confined in a cylindrical Penning-Malmberg trap of radius R_0; we will assume perfect thermodynamic equilibrium. Let us denote the applied electrostatic potential as V_0 and the axial magnetic intensity as $\mathbf{B}$. A rotating wall electric quadrupolar field of intensity $\lambda = 1/2 + \delta$ and angular frequency ω is then added which prompts the plasma cloud to rotate collectively with the same rotating wall angular frequency ω. The latter is the guiding orbit magnetron frequency of the ion cloud. The charged plasma remains confined in the central plane of the trap and the form of the confined ion cloud is a spheroid of semi-major axis R_C and semi-minor axis z. Using the fact that ions should be in equilibrium inside the confined cloud, it is easy to see that the electric axial oscillations of ions should be related to the volume density number,n, of this cloud as $\omega_z = (ne^2/m\epsilon_0)^{1/2}$. Moreover, the oscillator axial frequency also depends on the electric potential V_0 and the radius of the trap cylinder, R_0, as $\omega_z = (2/R_0)\sqrt{eV_0/m}$. In addition, ought to the applied axial magnetic field $\mathbf{B}$, the ions inside the confined spheroid have an internal fast cyclotron oscillation whose frequency is defined in terms of $\mathbf{\Omega} = e\mathbf{B}/m$. Due to the diamagnetic behaviour of charge currents inside the plasma cloud, the achievable cyclotron frequency of the Deuterons is actually smaller: $\Omega' = \Omega - 2\omega$, which this is called the vortex frequency. On the other hand, dynamic equilibrium of the plasma imposes that $\omega(\Omega - \omega) \to \omega_z^2/2$. This means that there exists a key parameter ϑ to define confinement quality of the Penning-Malmberg trap such that $\omega = \Omega \sin^2 \frac{\vartheta}{2}$ and $\omega_z = \Omega \sin\vartheta/\sqrt{2}$. When $\omega_z \ll \Omega$ the aspect ratio satisfies the condition $\ell = z/R_C \ll 1$ (see Ref. [10]). Ion confinement is expected to be perfectly stable free of the magnetohydrodynamic instabilities that happen in magnetic neutral plasma confinement. This is the confinement theorem of charged plasmas. In special situations, there are other kind of exact periodic solutions of the equations of motion (see Appendix). These solutions represent the trajectories of two correlated confined nuclei that follow coalescing paths to the center of the trap, where they collide, and so D-D fusion reactions will happen. Yet, this effect will hold only with a given probability, since most of the ions in the cloud will still follow circular magnetron guiding orbits. Thus, the correlation only occurs if the special condition between the δ parameter and the ϑ trap parameter holds, which is the ion resonant confinement condition. To see this, recall that, in the rotating center of mass frame of every two ions, the trap field interaction Lagrangian is given in terms of the relative distance ϱ between the correlated nuclei as $\mathfrak{L} = \frac{1}{2}\mu\dot{\varrho}^2 - U(\varrho, t)$, the potential energy being $U(\varrho, t) = \frac{1}{2}\mu\omega_z^2\varrho^2(\lambda\cos 2\omega t - \frac{1}{2})$, where $\mu = m/2$ stands for the reduced mass of the ions. It is seen that the fast cyclotron motion frequency does not appear in $U(\varrho, t)$. The solution of the equation of motion of the two-correlated-nuclei complex is given in terms of Mathieu functions. If R_C is the maximum radius of the orbit, denoting, $\chi = \cot^2(\vartheta/2) \to \frac{eB^2R_0^2}{4\mu V_0} - 2$ (for $\vartheta \ll 1$), one gets,

$$\varrho = \frac{2R_C}{q_0} C_e[-\chi, -\lambda\chi, \omega t\,], \tag{2}$$

where q_0 is a normalization constant. In general, these orbits are exponentially unstable, yet numerical calculations show that there exists a special, resonant,π/ω periodic orbit whenever the quadrupolar field strength parameter δ takes the value, (see Appendix)

$$\delta \equiv \lambda - \frac{1}{2} \to \frac{1}{\sqrt{2\chi}}. \tag{3}$$

The minimum distance between the nuclei in these resonant orbits will eventually be given by the following numerical solution

$$\varrho_0 = 2R_C\kappa(\chi), \text{ with } \kappa(\chi) \simeq \exp\{1/2 - 1.35\sqrt{\chi}\}, \tag{4}$$

which can reach arbitrarily small values at any temperature of the confined plasma in the limit $\vartheta \to 0$. Moreover, the kinetic energy per Deuteron in this case becomes $\mathcal{E} \to W/2$ where

$$W = \frac{e^2}{4\pi\epsilon_0\varrho_0}. \tag{5}$$

Which provides $\varrho_0(W)$. The exact trajectories of two correlated ions colliding at the center of the trap vessel is shown in Fig. 1.

On the other hand, it has been shown in the Appendix that a charged plasma in thermodynamic equilibrium at temperature T should behave as a solid rotor in a Penning-Malmberg trap with a magnetron frequency oscillatory stroboscopic rotating wall quadrupolar field. The plasma global angular velocity ω , coinciding with that of the individual ions guiding orbits, satisfies the condition $\omega = \sqrt{k_BT/\mu R_C^2}$. Then, since $\omega = \Omega/(\chi+1)$, we

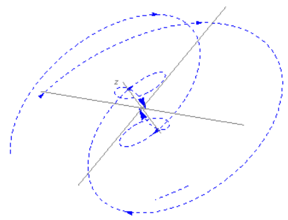

FIG. 1. A simulation of the correlated trajectories of two ions when the resonant conditions of the trap are satisfied. The two deuterons collide at the center of the trap.

get $\chi+1 = \Omega R_c\sqrt{k_BT/\mu}$ which, owing to the dynamic internal equilibrium of the plasma, it is equivalent to saying that $\chi \simeq eV_0/(k_BT)R_c^2/R_0^2$. Finally, using this thermodynamic constraint and given that the axial degree of freedom is thermal, i.e., $k_BT = 1/2\mu\omega_z^2 z_{max}^2$, we conclude that the aspect ratio of the confined ion cloud minimal height spheroid is $z_{max}/R_c \simeq 1/\sqrt{\chi}$, which is an important relation that will be used later. In the Appendix, it has been also demonstrated that the probability that two correlated ions follow the resonant orbits is $\wp(\chi) = 1/\chi$. Recall that in the resonant case, all the macroscopic features of the Deuteron plasma can be written only in terms of the microscopic parameter W and of the trap confinement value for χ. Then, $R_c = e^2/(8\pi\epsilon_0)W^{-1}\kappa(\chi)^{-1}$ and the confined volume of the ion cloud becomes

$$V \to \tfrac{4}{3}\pi(R_c/\sqrt{\chi})^3 \times \chi, \tag{6}$$

which is coincident with the volume of χ ion clump spheres of radius $R_c/\sqrt{\chi}$. The density number becomes

$$n = \frac{\epsilon_0 B^2}{4\mu}\sin^2\vartheta \to 4\omega^2\epsilon_0\mu\chi/e^2$$

and the total number of confined Deuterons $N \to V \times n$.

III. FUSION REACTION POWER BALANCE.

For Deuterium, the natural occurring reactions are

$$\begin{aligned} {}^2\mathrm{H} + {}^2\mathrm{H} &\to {}^3\mathrm{He} + \mathrm{n}, \\ {}^2\mathrm{H} + {}^2\mathrm{H} &\to {}^3\mathrm{H} + {}^1\mathrm{H}, \end{aligned} \tag{7}$$

their corresponding fusion energies are $E_{f1}[{}^3\mathrm{He}+\mathrm{n}] = 3.27$ Mev and $E_{f2}[{}^3\mathrm{H} \ +{}^1\mathrm{H}] = 4.04$ Mev. Each reaction takes place with approximately 50% probability. Thus, together with the necessary Lawson criterium in Eq. 1, in a self-sustaining energy balanced fusion Deuterium device the only heating terms shall be that from the kinetic energy of the confined charged products in Eqs. 7. The kinetic energy contribution of the neutrons, which cannot be confined in the trap, must be subtracted from the nuclear energy balance, a fraction $\eta_n = E_n/(E_{f1}+E_{f2})$. The neutron energy escapes directly to heat the reactor walls. On top of that, other energy losses have to be taken into account in the power balance analysis, namely the Bremsstrahlung term due to accelerated charges in the plasma, as well as that corresponding to ion-ion Coulomb collisions, or the transport term. Positive power balance of the deuteron plasma, then, requires

$$\mathcal{P}_f(1-\eta_n) > \mathcal{P}_B + \mathcal{P}_L, \tag{8}$$

where $\mathcal{P}_f \equiv \mathfrak{I}_f V$ is the power due to fusion reactions that heats the whole plasma volume V, $\mathcal{P}_B$ is the *Bremsstrahlung* term and $\mathcal{P}_L$ is the power loss caused by transport, assumed collisional. Again, let $\mathcal{E}$ be the energy of the deuteron nuclei in the resonant orbit (this correlates with the effective temperature of the nuclei *inside* the density clusters) and let $n'(\chi)$ be the density number of the plasma in the resonant clusters, then the collision frequency is, (see Ref. [9]) $\nu(\chi,\mathcal{E}) = n'(\chi)\mathcal{E}^{-3/2}\frac{3e^4}{16\pi^2\epsilon_0^2 m^{1/2}}\ln\Lambda(\chi,\mathcal{E})$. The Coulomb logarithm is $\ln\Lambda(\chi,\mathcal{E}) = \ln\left\{12\pi\epsilon_0^{3/2}n'(\chi)^{-1/2}e^{-3}\mathcal{E}^{3/2}\right\}$. Now, we write $N' = \wp(\chi)N$ as the number of resonant nuclei in the density clusters; then $V' = \kappa(\chi)V$ is the volume of the density clump because only one of the two major axis of the spheroid is compressed due to the resonant orbits of the nuclei in it. Then every ion swirls around the center in a vortex from $r = R_c$ to $r = \varrho_0/2$ and, therefore, the effective volume that affects all the resonant orbits may be approximated as $V' = 4/3\pi R_c\left\{\kappa(\chi)R_c\right\}z$, which will be the volume available for fusion reactions in the center of mass of the resonant complex. Then, $n'(\chi) = \wp(\chi)/\kappa(\chi)n$. Now, from these definitions, the reactor effective power per unit of volume becomes

$$\begin{aligned} \mathfrak{I}(\mathcal{E},\chi) = (1-\eta_n)\mathfrak{I}_f(\mathcal{E};\chi) - \\ -\eta\left\{n'(\chi)^2\alpha_B\mathcal{E}^{\frac{1}{2}} + \frac{N'}{V}\mathcal{E}\nu(\chi,\mathcal{E})\right\} \end{aligned} \tag{9}$$

where the fusion power per unit volume is

$$\mathfrak{I}_f(\mathcal{E};\chi) = \left\{\tfrac{1}{2}E_{f_1}\langle\sigma v\rangle_1 + \tfrac{1}{2}E_{f_2}\langle\sigma v\rangle_2\right\}(\mathcal{E})\frac{n'(\chi)^2}{2}. \tag{10}$$

Notice that in Eq. (9), we consider the total volume of the plasma in the collision term, instead of that of the resonant cluster of ions; the reason is that we consider the most unfavourable case in which the collisional transport is large enough to drive the local inhomogeneities that appear in the resonant zone to the entire plasma volume. In Eq. (9) $\eta \approx 1$ because, neglecting the ^{3}H, ^{1}H and ^{3}He concentrations, i.e., those of the nuclear reaction products, there is only a single particle species in the confined plasma. For long period working of the reactor, the concentrations of these species could not be negligible and should be taken into account. In Eq. (9), $\alpha_B = 1.4 \ 10^{-40} \ (m_e/m_p)^{3/2} \quad [\mathrm{W}/m^3K^{-1/2}]$ and, for $\mathcal{E}(keV) < 300$, we can make the following analytical approximation for the cross-section of the reaction: $\langle\sigma v\rangle_i = b_i\mathcal{E}^{\beta_i}\exp\{c_i\mathcal{E}^{\chi_i}\}$ $b_1 = 1.198\times10^{-18}$,$b_2 = 3.5501\times10^{-19}$,$\beta_1 = -1.0759$, $\beta_2 = -0.9462$, $c_1 = -23.511$, $c_2 = -22.04$, $\chi_1 = 0.29221$, $\chi_2 = 0.2922$. Using these values in Eq. (9) one sees that the Bremsstrahlung and the ion collision terms have small effects in the power balance.

IV. THERMODYNAMIC CONSTRAINTS.

In order for the Cell to be energetically self-sustained, the generated energy must be greater than the sum of the emitted heat plus the magnetic back reaction pressure loss (as required from Brillouin's theorem). Then, the actual thermodynamically available power density is

$$\mathfrak{I}_C = \mathfrak{I} - \dot{\mathcal{W}}, \tag{11}$$

where $\dot{\mathcal{W}} = \mathfrak{b}^2/2\mu_0\omega(\chi)/\pi$ is the power loss per unit of volume due to the diamagnetic currents of the confined ions calculated for the resonance frequency. Recall that the contribution to the pressure losses is written in terms of the plasma back reaction magnetic field, namely, $\mathfrak{b} = -4(\mu/e)\omega = -2/(1+\chi)\mathbf{B}$. Now, the energy confinement time τ_E may be calculated in a Cell of temperature T and density number n for the nuclei in the resonance, whose density number becomes $n'(\chi)$ near the ignition point: $\tau_E[\text{Cell}] = n'(\chi)\mathcal{E}/\mathfrak{I}_{\text{C}}$. Also, in the Cell, the pressure at the ignition point becomes $p[\text{Cell}] = 2/3n'(\chi)\mathcal{E}$. Then, (see Ref. [9])

$$p\tau_E[\text{Cell}] = \tfrac{2}{3}n'(\chi)^2\mathcal{E}^2/(\mathfrak{I}(\chi;\mathcal{E}) - \dot{\mathcal{W}}). \tag{12}$$

The function $p\tau_E[\text{Cell}]$ should have a minimum value for some $\chi(\vartheta)$ and $\mathcal{E} = W/2$. This can be seen in Fig. 2 where Lawson's minimum is actually reached for $\vartheta \leq 0.105$, and $W = 30.7$ keV.The minimum is obtained for approximately the same Cell parameters, independently of the actual temperature of the confined plasma cloud. Thus $p\tau_E[\text{Cell}] \approx 130$ atm · s , which is expected to be experimentally achievable. The value of the Coulomb barrier energy for the Lawson minimum computed for the Cell is remarkably close to the expected experimental value (see Ref. ([9]), a fact that supports the Resonant Ion Confinement reactor concept. Now, in order to obtain the actual power efficiency of the Cell, recall that, since the species $^3\text{He}_2$, $^1\text{H}_1$ and $^3\text{H}_1$ are positively charged, they will be retained in the trap but, contrarily to that, the neutrons will escape from the trap cavity and will be absorbed by the surrounding walls of the Cell, where a system to absorb their energy should be installed. Their kinetic energy might be, then, transformed into heat (with an efficiency $\eta_h \sim 1/3$). This heat can used to produce electricity by means of high efficient thermoelectric materials.

Moreover we must also take into account that every D-D collision takes place by quantum tunnelling the barrier W. The Gamow-Sommerfeld probability of this transition needed to undergo the nuclear reaction is $\eta_f(W) \sim \exp\{-\pi\alpha c\sqrt{2\mu/W}\}$, again, the Deuteron reduced mass is $\mu = m/2$. It gives, for the Lawson minimum $W \simeq 30.7$ keV, $\eta_f \simeq 1/300$. Having this in mind, the following estimate for the electric power of the Cell can be provided $P_f \sim \eta_n\eta_h\mathfrak{I}_C \times V(W,\chi) \to \eta_n\eta_h\mathfrak{I}_C \times \{\mathfrak{N}V'(W,\chi)\}$, where we have used the fact that the real available volume of the reactants in the Cell must be calculated considering that the number of reactions in the total volume of the Cell should match, in the average, to the sum of all

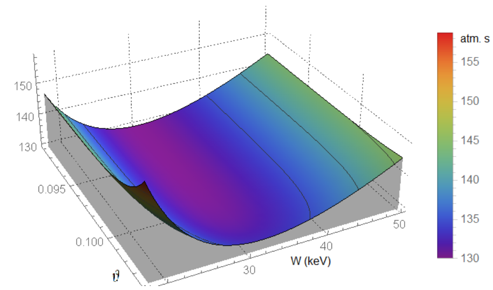

FIG. 2. Lawson triple product $p\tau_E[\text{Cell}]$ for $B = 3.73$ T as a function of the Coulomb barrier energy achievable for D-D collisions W in keV and the parameter $\vartheta = \arcsin(\frac{\sqrt{2}\omega_z}{\Omega})$. This minimum exists only for $\vartheta \lesssim 0.105$ and $W \sim 30.7$ keV.

tunnelling-through-the-barrier collisions. This is given by the quantity $\mathfrak{N} = \eta_f(W) \times \wp(\chi)^2 \times N^2/2$. The following equation gives the key constraints for the Cell confinement configurations

$$\wp(\chi)^2 N^2/2 \times \eta_f(W)[\kappa(\chi)V] = V. \tag{13}$$

Also, the operative condition

$$R_c = \frac{1}{2}\varrho_0(W)\kappa(\chi)^{-1} \leq \frac{2}{eB}\sqrt{eV_0\mu(\chi+1)} = R_0, \tag{14}$$

must be fulfilled. The total thermoelectric power of the Cell becomes

$$\mathbf{P}_{Cell} = P_f - (1-\eta_h)aT^4[\pi/\omega(\chi)]V. \tag{15}$$

Here, to obtain the realistic model of the Cell, we have subtracted the black body radiation term (a is the radiation constant). In addition to that, the maximum variation of the number of Deuterons per unit of time in the Cell is $\dot{N}_{\text{D}} = \frac{1}{\eta_n}P_f/\langle E_f\rangle$ and, therefore, the Cell thermonuclear reaction frequency is $\nu_f = \dot{N}_{\text{D}}/N$. For the actual Fusion Cell, remember that the Deuteron fuel will enter the Cell cavity in a pulsed way. These fuel pulses react in a time of $1/\nu_f$, to this point, the fuel is introduced in the reactor chamber through very high frequency pulses and the achievable electric power of the Cell will depend on the ultra high frequency and high voltage circuit breakers to control the rapid pulsed fuel refilling of the reactor chamber. Then, if we denote the frequency of these circuit breakers as ν_{CB} , the achievable power of the Cell would be: $\nu_{\text{CB}}/\nu_f\mathcal{P}_{[Cell]}$. A solution of the Cell constraint in Eq. 13 does exist and is $\chi = 477.102$, $B = 3.73$ T, and $W = 30.74$ keV. The resulting maximum achievable electric power of this Cell would be

$$\mathbf{P}_{\text{Cell}} \simeq 4.4 \text{ MW}$$

for Deuteron pulses of $N = 2.2 \times 10^{10}$ injected with a frequency $\nu_f \sim 5.65$ GHz when the electric potential is within the range 5.9 kV $< V_0 <$ 8.5 kV.

A solution of Eqs. 13 and 14 is represented in Fig.3 for the attainable thermoelectric power in Eq. 15; this configuration gets its maximum at $V_0 \simeq 6.5$ kV for a fuel rate of $\dot{N}_D \simeq 10^{20}$ Deuteron/s. The confinement radius of the ion cloud would be $R_c \simeq 91$ mm and the trap radius is

$R_0 \simeq 92.56$ mm. Of course a realistic facility could reach several orders of magnitude less electric power because the actual Cell requires a pulsed refilling of the reactor cavity and that the resonant configuration would be attainable only adiabatically, i.e., $\lambda \to 1/2 + \delta$ (after Eq. 3) slowly enough in a time much larger than ν_f^{-1}. This kind of technological requirements will be the subject of the experimental research for real fusion facilities based on the grounds of the resonant ion confinement concept.

Yet, in order to double check that the above estimates are correct, we can do a rapid analysis. As said, in the resonant cell (all over the time), the number of possible configurations between two Deuterons, leading to quantum tunneling reactions, is approximately $\mathfrak{N} \sim \eta_f(W)\wp(\chi)^2 \times N^2/2$. Since the natural colliding frequency is $\Omega/2\pi$, the nuclear reaction rate should be $\mathfrak{N}\times\Omega/2\pi$, and the number of emitted neutrons per second is half this value. Then, roughly, the actual achievable electric power must be $\mathbf{P}_{\rm Cell} \sim \eta_h 2.44\ \mathrm{MeV} \times \frac{1}{2}\mathfrak{N}\times\Omega/2\pi$. For the numbers above, i.e., $B = 3.7$ T, $N = 2\times10^{10}$ Deuterons, $\chi \sim 477$, and $R_C \sim 91$ mm (which, for the resonant condition in Eqs.4, is equivalent to saying that $\varrho_0 \simeq 47$ fm or $W \simeq 30.7$ keV), we get $\mathbf{P}_{\rm Cell} \sim 6$ MW, which is of the order of the figure that we did obtain from nuclear theory alone. This fact makes us confident in the correctness of the derived result.

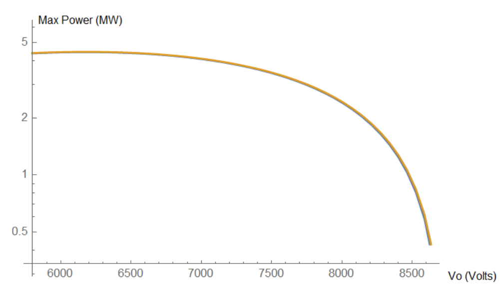

FIG. 3. Optimal achievable power of the standard Cell, $\mathcal{P}_{[Cell]}$ as a function of the applied confinement electric potential V_0, $B = 3.73$ T and $\vartheta = 0.0915$.

V. DISCUSSION

For a compact reactor of small size, a new type of fusion technology can be developed according to the basic conditions described in this article, however, some practical considerations must be taken into account. First recall that this possibility arose from imposing, to the standard Penning-Malmberg ion trap, the parametric relations in Eqs. 3 and 4, which lead to the necessary resonant kind of ion confinement. On the other hand, the fusion conditions of the trap are the key configurations satisfying the system in Eqs. 13 and 14 and, very promising and satisfactorily, both, the resonant and the operative conditions for Deuteron confinement, are found, precisely, within the range of the current state of the art technological capabilities. In other words, due to the resonance the Coulomb barrier does not appears as an obstacle for fusion. Additionally, one must also expect that a pure Helium-3 Fusion Cell will work in exactly the same principles than the Deuterium one exposed in this article. Recall such Helium-3 Cell will be aneutronic and, as a safe reactor, it opens the possibility to implement a sort of thermonuclear battery in the event that it could power an autonomous device provided with some finite reservoir of Helium-3 gas. Notwithstanding with this, even though the physics of both types of cells should be very alike, it does not necessarily imply that their engineering designs, which have to be associated with solving the problems of each type of reactors, should be also, in turn, similar. Therefore the Helium-3 Fusion Cell will be studied separately.

A second consideration is that, since the fuel enters the trap chamber in a pulsating manner, experimental analysis must be performed to determine the best circuit breaker frequencies required to provide the ion resonance stability conditions after the fuel has been released and once the initial power-up has been done, i.e., most likely, in practical situations, some thermal relaxation time will be needed to regulate the ignition cycles. Third, and no less important than the power cycle problem, is the resonant configuration time scale problem that arises because the resonance must be obtained after adiabatically adjusting the intensity of the rotating wall to the resonant one, an operation which must be achieved in times much greater than that corresponding to the speed of the fusion reaction (which can be estimated in the order of ns). These Fusion Cell engineering issues remain to be elucidated.

VI. ACKNOWLEDGEMENTS.

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Appendix A

The ion confinement of Deuterium nuclei in a cylindrical Penning-Malmberg trap, with a rotating wall electric quadrupolar field, will be reviewed to find that, together with standard magnetron trajectories with rapid cyclotron rotation, there are coalescing orbits towards the center of the trap, which are also compatible with the cyclotron degree of freedom. The second class of trajectories will become very probable when special resonance conditions are given for the intensity of the quadrupolar field. This possibility is very promising since, contrarily to the case of neutral plasmas, the confinement stability of positively charged plasmas is warranted and no Magneto-Hydrodynamical instabilities are expected.

A cylindric Penning Malmberg trap is characterized by the introduction of an axial magnetic field $\mathbf{B}$ and a electrostatic potential V_0, in a cylindrical cavity where the ions are introduced. Additionally the full stability can be obtained with the help of a rotating electric field with angular frequency ω (see Ref. [10]) and field strength λ; in this case, the motion of the ions in the trap is decomposed into separated rotationally confined mode and an axial oscillation. Let q be the charge of the ion, then, the Lagrangian is given in terms of the electrostatic quadrupole and the magnetic field frequencies of the trap, i.e., denoting the cyclotron frequency $\Omega = qB/m$

$$\mathcal{L}=\sum_i \tfrac{1}{2}m\{\dot{x}_i^2+\dot{y}_i^2+\dot{z}_i^2 + \Omega(x_i\dot{y}_i-y_i\dot{x}_i)\}-U_t(\mathbf{r}_i), \quad \text{(A1)}$$

$U_t(\mathbf{r})=qV_0+\frac{1}{2}m\{\omega_z^2[z^2-\frac{1}{2}\rho^2]+\lambda\omega_z^2[(x^2-y^2)\cos 2\omega t-2xy\sin 2\omega t]\}$, and $\rho^2=x^2+y^2$. This rotating quadrupolar electrical potential wall of intensity λ might be required for the adiabatic stability of the ions in the trap as it will be confirmed down from. Let us consider the rotating coordinate frame which rotates with angular frequency ω: $(x, y) \to (\xi, \zeta)$.

$$x = \xi\cos(\omega t) + \zeta\sin(\omega t),$$
$$y = -\xi\sin(\omega t) + \zeta\cos(\omega t).$$

Then the rotating wall quadrupole perturbation becomes time independent,

$$(x^2 - y^2)\cos(2\omega t) - 2xy\sin(2\omega t) \to \xi^2 - \zeta^2, \quad \text{(A2)}$$

$$\mathcal{L}=\sum_i \tfrac{1}{2}m\{\dot{\xi}_i^2+\dot{\zeta}_i^2+\dot{z}_i^2 + (\Omega-2\omega)(\xi_i\dot{\zeta}_i-\zeta_i\dot{\xi}_i)\}-U'(\mathbf{r}_i), \quad \text{(A3)}$$

$U'(\mathbf{r})=qV_0+\frac{1}{2}m\omega_z^2z^2-\frac{m\rho^2}{4}\omega_z^2+\frac{m\rho^2}{2}\Omega\omega-\frac{m\rho^2}{2}\omega^2+\frac{m}{2}\lambda\omega_z^2(\xi^2-\zeta^2)$.

The constant energy potential term qV_0 will be omitted in the rest of this supplement.

The z coordinate motion is an harmonic oscillator of frequency ω_z. The restoring force gives the effective axial component of the electric field $eE_z = -m\omega_z^2 z$. Along these lines, if the density of the plasma cloud is n, E_z can be thought as that coming from the displacement of a positive charge in the plasma cloud (effectively a relative negative charge) $\Delta q = -qz\oint dSn$. Then, $\oint E_z dS = \Delta q/\epsilon_0$ and from this the charge density becomes

$$n = \frac{m\omega_z^2\epsilon_0}{q^2}. \quad \text{(A4)}$$

In order to compensate the axial displacement of every ion, this motion should be correlated with that of some other ion in the opposite z coordinate; moreover this z axis degree of freedom can be assumed to be thermal and, therefore, for these two correlated ions

$$2 \times \frac{1}{2}k_BT = \tfrac{1}{2}\mu\omega_z^2 z_{\max}^2. \quad \text{(A5)}$$

On the other hand, thermodynamical equilibrium makes sense if there are also microscopic collisions between pairs of ions providing the appropriate internal Coulomb field screening at the Debye length $\lambda_{\mathrm{De}}=(\epsilon_0 k_{\mathrm{B}}T/q^2n)^{1/2}$. Along these lines, plasma density fluctuations will appear. For those ion aggregates, as it can be seen from inspecting the Lagrangian in the quadrupole stroboscopic rotating frame, the trajectories should be determined by a guiding orbit of angular velocity ω plus a rapid cyclotron oscillation of reduced frequency $\Omega' = \Omega - 2\omega$. To such a degree to avoid dynamical plasma instabilities every charge fluctuation in the plasma should appear at two diametrally complementary positions (this fact is also warranted owing to the cylindrical symmetry of the system).

In order to determine the exact orbits of what may represent those complementary bundles of ions, let us first recall that in the rotating frame the net quadrupole radial force can be averaged out to zero in every cycle ($\langle F_{\mathrm{q}}\rangle = -\lambda m\omega_z^2\langle\rho(t)\cos 2\omega t\rangle \to \sim 0$, for $0 \le \omega t \le 2\pi$). Indeed, three forces are acting on each of the ions, namely, the centrifugal force $+m\omega^2\rho$, the radial electric force $+\frac{1}{2}m\omega_z^2\rho$ and that one associated with the radially electric field induced by the rotation ω through the direction of the axial magnetic field whose value is $-m\omega\Omega\rho$. It is this field that will provide the radial confinement. To see how, recall that, from the symmetry of a cylindrical trap, the total angular momentum (that from the ions plus that from the magnetic field) $L=\sum_i\{m_i v_{\theta i}\rho_i + qB\rho_i^2/2\}$ is preserved and, therefore, for instance for large B, $\sum_i \rho_i^2$ must reach some constant, which, as said, implies radial confinement; this argument is due to O'Neil (Ref. [11], Ref. [12]). With that in mind, the simplest case where the perturbed quadrupole frequency should be taken is the one where the net radial force acting on each of the charges is zero. Then,

$$\omega_z^2=2\omega(\Omega - \omega). \quad \text{(A6)}$$

It means that a "trap angle" can be defined such that

$$\omega_z^2 = \frac{1}{2}\Omega^2\sin^2\vartheta, \;\; \omega = \Omega\sin^2(\tfrac{\vartheta}{2}). \quad \text{(A7)}$$

Eq. A6 corresponds to the confinement situation of a thin, rigidly rotating, spheroid of plasma at angular velocity ω (Ref. [13] Ref. [14]). Recall also that for that confined cloud of ions of mass m and charge $q = Ze$ a relation between the axial frequency ω_z , the cylindric radius R_0 and the applied electric potential V_0 can be found as the solution of $V(R_0) = 0$, where, according to Eq. A4, $V(R) = V_0 - \frac{m}{4}\omega_z^2R^2/q$. This gives, $\omega_z^2 = \frac{4qV_0}{mR_0^2}$,

Lagrange's equations read (denote $\tau(t)=\Omega' t$ and $\varepsilon=\lambda\omega_z^2/\Omega'^2$)

$$\frac{d^2\xi}{d\tau^2} - \frac{d\zeta}{d\tau} + \xi\varepsilon = 0, \quad \frac{d^2\zeta}{d\tau^2} + \frac{d\xi}{d\tau} - \zeta\varepsilon = 0. \quad \text{(A8)}$$

Elseways, since the Lagrangian is not time dependant, the energy is preserved and a first integral of motion is obtained $E=\sum_i\{\dot{\xi}_i\partial_{\dot{\xi}_i}L+\dot{\zeta}_i\partial_{\dot{\zeta}_i}L\}-L=\sum_i \frac{m}{2}\{\dot{\xi}_i^{\,2}+\dot{\zeta}_i^{\,2}+\lambda\omega_z^2(\xi^2-\zeta^2)\}$. This means that the motion is bounded and that a time period t_0 can be found such that $2E/(\Omega'^2 m)=\varepsilon\sum_i\{\xi_i(t_0)^2-\zeta_i(t_0)^2\}$.

As said, the orbits of the ions are combinations of rapid bare cyclotron Ω' oscillations plus a slow guiding center magnetron trajectory of angular velocity ω which becomes stabilized by the rotating quadrupole force.

Notwithstanding with this, other solutions can be seen as representing binary collisions at the center of the trap.

Define, $\eta{=}\sqrt{1+4\varepsilon^2}$, $\sigma{=}\sqrt{1/2(\eta-1)}{\to}\varepsilon$, $\gamma{=}\frac{\varepsilon+\sigma^2}{\sigma}{\to}1$, $\beta{=}\sqrt{1/2(\eta+1)}{\to}1$ and $\gamma'{=}-\frac{\beta}{\varepsilon+\beta^2}{\to}-1$, for $\varepsilon{\ll}1$. With this notation the exact Eq. A8 solutions are

$$\begin{aligned} &\xi_{\rm h}{=}a\cosh\sigma\tau, \quad \zeta_{\rm h}{=}a\gamma\sinh\sigma\tau, \\ &\xi_{\rm C}{=}b\cos\beta\tau, \qquad \zeta_{\rm C}{=}b\gamma'\sin\beta\tau. \end{aligned} \tag{A9}$$

The $\xi_{\rm C}$, $\zeta_{\rm C}$ should correspond to the mentioned bare cyclotron orbits (see e.g., Hasegawa et al. Ref. [10] for the exact solutions of the cyclotron orbits in the rotating wall trap configuration). Thus, for instance, if the general solution is $\vec{r}=\vec{r}_h+\vec{r}_C$, the limit $\varepsilon\to 0$ is just the usual Penning trap constant radius magnetron orbit $|\vec{r}|=a$ provided with a rapid cyclotron oscillation around this guiding orbit.

Incidentally, in spite of the fact that $\xi_{\rm h}$, $\zeta_{\rm h}$ can be seen as spurious solutions (they are not bounded hyperbolae that do not meet the required confinement conditions), recall that, admissibly, some of the actual orbits could also be represented by linear combinations of these hyperbolic plus the bare cyclotron solutions:

$$\xi^{(i)}{=}\xi^{(i)}_{\rm h}+\xi^{(i)}_{\rm C}, \quad \zeta^{(i)}{=}\zeta^{(i)}_{\rm h}+\zeta^{(i)}_{\rm C}, \tag{A10}$$

trajectories that would likely exist during some period of time t_0, say, $-t_0{\le}t{\le}t_0$, .We will take this for granted and now let us be concerned with identifying the physical situations that these orbits represent.

Then, the numerical simulation shows that, in the rotating frame, the trajectory is just an hyperbola provided with rapid cyclotron oscillations having two turning points (at which $\dot{\xi}(t_0){=}\dot{\zeta}(t_0)=0$.) Is obvious that the physical situation corresponds to the orbits of two long range coupled ion density fluctuations that collide (and repel each other) at the center of the trap with $\xi^{(1)}{=}{-}\xi^{(2)}$, $\zeta^{(1)}{=}{-}\zeta^{(2)}$. This is obviously correct because, otherwise, single ions would not preserve linear momentum individually. The situation is shown in Fig. 4.

For two correlated ions clumps, these hyperbolic trajectories are perfectly valid solutions from Eq. A3 which, indeed, preserving the rotational symmetry of the problem, may be seen as a sum of terms that can be grouped and integrated for every two arbitrarily large clumps coordinates separately. On the other hand, the physics that this Lagrangian describes is just the diamagnetism of the charged plasma cloud, i.e., the situation in which the internal plasma currents modify the intensity of magnetic field. This diamagnetic behavior can be deduced from the fact that, in the rotating frame, the field intensity felt by the ion becomes $B\to B-2m\omega/e$, which mathematically corresponds to the Coriolis-like shift of the cyclotron frequency seen in Eq. A3, i.e., $\Omega\to\Omega-2\omega$. This means that the collective properties of the plasma will also hold in this case (including Brillouin's theorem for the limit of the number density of the ion plasma cloud). Yet, every coordinated two ions hyperbolic orbit will have a junction point at the center of the trap, thereby largely increasing, locally and instantaneously, the density at that point, a fact that shall not be regarded as a contradictory statement. To avoid this, it must be assumed that the coalescing orbits of the ions occur with some probability, and, therefore, the question of whether there is any method that stabilizes this probability must be studied, in other words, what the special conditions for the trap parameters should be leading the plasma to acquire a new collective state, in which the deuterium nuclei will collide naturally in the center at a predictable rate. In this investigation one could be guided from some dynamical analogies. Take, for instance, the case of an inverse pendulum with some forced time periodic term. In this case, by varying the intensity of the time periodic force, depending on the length of the pendulum, some solutions can be found that renders the inverse pendulum stable. Following exactly this analogy, one guess that some special relation between the trap frequencies and the quadrupole field parameters, ω and λ, will be required in order to render the plasma to a new regime of stability in such a way that the probability rate of the hyperbolic orbit corresponding to two correlated ion clumps be predictable. Due to the fact that collectively the plasma should preserve the internal electro-dynamical equilibrium some dynamical collective motion transition for the plasma should also occur.

Let us assume that this situation does indeed exist; we call this as the Resonant Ionic Confinement solution first introduced in [7]. The only relevant degree of freedom ought be the relative distance between the two charged aggregates, $\varrho{=}2(\xi^2{+}\zeta^2)^{1/2}$. For this coordinate, the collision is described from the Lagrangian Eq. A3 replacing $\xi{=}\varrho\cos\omega t$, $\zeta{=}\rho\cos\omega t$, and $m{=}\mu{=}m/2$

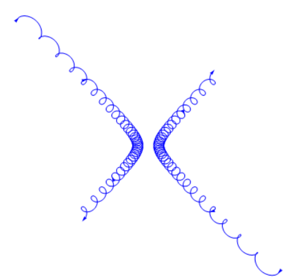

FIG. 4. Two ion density clumps collision at the center of a cylindrical Penning trap.

$$\mathcal{L}{=}\tfrac{1}{2}\mu\{\dot{\varrho}^2+\tfrac{1}{2}\omega_z^2\varrho^2-\omega_z^2\varrho^2\lambda\cos 2\omega t\}. \tag{A11}$$

The Energy functional reads

$$\mathcal{H}{=}\tfrac{1}{2}\mu\{\dot{\varrho}^2-\tfrac{1}{2}\omega_z^2\varrho^2+\omega_z^2\varrho^2\lambda\cos 2\omega t\}. \tag{A12}$$

For the periodic orbits that we are looking at it is possible to calculate their average energy which should be some constant of motion

$$\langle\mathcal{H}\rangle\propto\tfrac{1}{2}\mu\dot{r}^2_{\rm max}-2qV_0(R_c/R_0)^2 \tag{A13}$$

Recall that this quantity refers to the radial part of the motion and that it does not include the contributions from the fast bare cyclotron mode. Denoting $\varphi=\omega t$ and $\chi=\frac{\omega_z^2}{2\omega^2}=\cot^2(\frac{\vartheta}{2})$ the equation of motion reads

$$\frac{d^2}{d\varphi^2}\varrho-\{\chi-2\lambda\chi\cos 2\varphi\}\varrho=0. \tag{A14}$$

Eq. A14 is Mathieu's equation whose time symmetric solution is obtained in terms of the Mathieu Cosine function:

$$\varrho(\varphi)=\tfrac{2R_{\mathcal{C}}}{c}C_e(-\chi,-\lambda\chi,\varphi),\quad \text{with } c=C_e(-\chi,-\lambda\chi,0).$$

Thus, as in the case of the inverse pendulum, there are periodic stable bound solutions only within a very narrow parametric region $\lambda(\chi)$ (see Ref. [15]), they have period π for the variable φ. For $\omega_z \gg \omega$, numerically, this parametric stability constraint corresponds to a dependency between the quadrupolar electric force intensity and the parameters of the Penning trap

$$\lambda\rightarrow\tfrac{1}{2}+1/\sqrt{2\chi} \tag{A15}$$

Additionally, the closest distance between the two ion bundles defines the squeezing factor, κ, of the coalescing orbits, i.e., numerically

$$\ln\{\varrho_0/2R_{\mathcal{C}}\}\rightarrow\ln\kappa=\tfrac{1}{2}-1.35\sqrt{\chi} \tag{A16}$$

here $\varrho_0\equiv\varrho(\pi/2)$ .The implication is that, if the resonant condition in Eq. A15 is satisfied, any closeness, even small, between the two positive ion clumps can be reached near the center of the trap when $\chi\gg 1$.

Eqs. A15 and A16 constitute the grounds of the Resonant Ionic Confinement Method.

Given that the motion is confined the average energy must be some constant and, taken this constant $\langle\mathcal{H}\rangle=0$ we get the dynamical constraint,

$$\tfrac{1}{2}\mu\dot{r}_{\max}^2-2qV_0(R_{\mathcal{C}}/R_0)^2=0, \tag{A17}$$

that will be useful later on.

Owing to the existence of the quadrupole, the magnetron degree of freedom is stabilized and the plasma is macroscopically described as a rigid rotor of angular velocity ω. Notwithstanding with this, microscopically, each individual ion radial velocity should be Maxwellian and the stability of the plasma requires that the solid rotor energy be thermal. Incidentally, the actual inertia momentum of every "rigidly rotating" stabilized thin disk of plasma becomes $I_{\rm Disk}=\sum_{i\in{\rm Disk}}m_ir_i^2=\frac{1}{2}N_{\rm Disk}mR_{\mathcal{C}}^2$, providing a rotational energy giving by $E_{\rm Disk}=\frac{1}{2}I_{\rm Disk}\omega^2$; yet, to this rotational degree of freedom of each individual ion, a thermal energy $k_{\rm B}T/2$ must be allocated. Along these lines, the condition of thermal equilibrium of the stabilized, rigidly rotating, plasma is compelled to be

$$N_{\rm Disk}\times\frac{k_{\rm B}T}{2}=\frac{1}{2}I_{\rm Disk}\omega^2.$$

Imposing that

$$\omega R_{\mathcal{C}}=\sqrt{k_{\rm B}T/\mu} \tag{A18}$$

which, since $\omega\simeq\Omega/(\chi+1)$, also implies, in the approximation $\omega_z\gg\omega$, that $\chi\simeq\frac{qV_0}{k_{\rm B}T}(\frac{R_{\mathcal{C}}}{R_0})^2$. Additionally, the density clumps in the coalescing orbits are practically confined in the center of the trap most of the time and they move away to reach some maximum confinement radius $R_{\mathcal{C}}\leq R_0$ where they bounce back again to the center. In this case, the ions inside those large aggregates, may interact individually when, owing to the resonance, the relative distance between the clumps is reduced to a tiny minimum. It is straightforward to derive, after Eqs. A4, A5 and A17, the following relations for $\chi\gg 1$

$$\begin{aligned}
B &\simeq \frac{2\chi}{qR_{\mathcal{C}}}\sqrt{k_{\rm B}T\mu}, && (a)\\
n &\simeq \frac{\epsilon_0}{\mu\chi}B^2, && (b)\\
z_{\max} &\simeq R_{\mathcal{C}}/\sqrt{\chi}, && (c)\\
N=n\times 4/3\pi R_{\mathcal{C}}^2 z_{\max} &\simeq n\chi\times V_c, && (d)\\
\dot{r}_{\max} &\simeq 2\sqrt{\frac{\chi k_{\rm B}T}{\mu}}=2\Omega\frac{R_{\mathcal{C}}}{\sqrt{\chi}}. && (f)
\end{aligned} \tag{A19}$$

where $V_c=V/\chi=\frac{4}{3}\pi(R_{\mathcal{C}}/\sqrt{\chi})^3$ is the clump volume. Therefore, we see, the confined plasma volume can be calculated as if there were χ granular spheres of ions of radius $R_{\mathcal{C}}/\sqrt{\chi}$. From these equation it is possible to derive that

$$n=\{(N/\chi)\,2\times\frac{1}{2}k_{\rm B}T/E_{\mathcal{C}}\}\times\{(N/\chi)/\tfrac{4}{3}\pi R'^3\}$$

where $R'=R_{\mathcal{C}}/\sqrt{\chi}$, whereas, $E_{\mathcal{C}}=(qN/\chi)^2/(8\pi\epsilon_o R')$, corresponds to the Coulomb energy of a bubble of N/χ ions on the surface of a sphere whose radius is precisely R'. Yet, recall that the plasma is fully thermal, imposing that the two dimensional surface energy satisfies $2\times\frac{1}{2}(N/\chi)k_{\rm B}T=E_{\mathcal{C}}$.

These thermal conditions of the plasma are, indeed, fully compatible with the existence of small fluctuations in the statistics that, according to our interpretation of the two coalescing ion density clumps, in the resonant case, will orbit the cloud at periodic Mathieu trajectories. For some clump pair in the plasma, the resonant situation will hold with some probability, say $\wp[i\in\{\text{coalescing}\}]$. To estimate this recall that, owing to the periodic radial displacement of the density bundles, analogously to the axially periodic degree of freedom, dynamical equilibrium imposes that there should be some effective radial restoring force $q\delta E_{\varrho=-\mu(2\omega)^2\delta\varrho}$. Again, this corresponds to a negative effective displaced charge (a hole in the positively charged ion cloud) of $-q\delta\varrho n'\oint dS$, where n' is the density of the displaced density clumps inside the plasma. Now, Gauss theorem applied to a thin disk surface of the plasma states that $n'=2m\epsilon_0\omega^2/q^2$, which, from its statistical definition $n'_{\equiv n\wp}$, and together with Eqs. A4 and A6, gives

$$\wp=\frac{\omega}{\Omega-\omega}=\chi^{-1}, \tag{A20}$$

which, given that $\omega_{<}\Omega/2$, it is always lower than 1 as it should be. Recall that near to the center of the trap, the minumum distance between the colliding ions can be approximated by $\varrho(t)\simeq|\varrho_0+2R_{\mathcal{C}}\varepsilon\cos\Omega' t|$, it means that the distance obtains its minimum when $\Omega'\Delta t_{=\pi}$.

[1] J.D. Lawson (1957),“Some criteria for power producing thermonuclear reactor”, Proc. Phys. Soc. B, 70, 7.

[2] A. J. H. Donné (2019), “The European roadmap towards fusion electricity”, Phil. Trans. R. Soc. A 377: 20170432

[3] K. Lackner, N. Gottardi (1990), “Tokamak confinement in relation to plateau scaling”, Nucl. Fusion 30, 767.

[4] A. Hassanein, V. Sizyuk. Hassanein, A., Sizyuk, V. Potential design problems for ITER fusion device”. Sci. Rep. 11, 2069 (2021). https://doi.org/10.1038/s41598-021-81510-2

[5] A. Dinklage et al. (2018), “ Magnetic configuration effects on the Wendelstein 7-X stellarator”, Nature Phys. 14, $855-860$.

[6] F. Castejón, A. J. Rubio-Montero, A. López-Fraguas, E. Ascasbar, and R. Mayo-García, (2016) “Fusion Science and Technology”, 70 pp. 406416.

[7] J.L. Rosales, and V. Martin, (2021) “A resonant ion confinement fusion reactor”. Spanish Patent Request No. 202131221, Universidad Politécnica de Madrid, (December 2021).

[8] T. Hasegawa, M. J. Jensen and J. J. Bollinger, (2005) “Stability of a Penning trap with a quadrupole rotating electric field”, Phys. Rev A 71, 023406.

[9] J. Freidberg (2007), “Plasma Physics and Fusion Energy”, Ed. Cambridge Uni. Press, ISBN $978-0-521-85107-7$. pp. 65, 111.

[10] T. Hasegawa, M. J. Jensen and J. J. Bollinger, (2005) “Stability of a Penning trap with a quadrupole rotating electric field”, Phys. Rev A 71, 023406.

[11] T.M. O’ Neil, “Plasma with a single sign of charge (an overview)”, (1995), Phys. Scripta T59 pp 341-351.

[12] O’ Neil, T.,(1980),“A confinement theorem for nonneutral plasmas”, Phys. Fluids, 23, 2216

[13] D. H. E. Dubin and T. M. O’Neil (1999) “Trapped nonneutral plasmas, liquids, and crystals (the thermal equilibrium states)”, Rev. Mod. Phys. 71, p87.

[14] J.J. Bollinger, D.J. Wineland and D.H.E. Dubin, (1994), Phys. Plasmas,1, “Non-neutral ion plasmas and crystals”, laser cooling, and atomic clocks, 1403

[15] L. Meirovitch, “Methods of Analytical Mechanics”, (1970). McGraw-Hill, reprinted by Dover Books, NY, (2003) ISBN 0-486-43239-4, pp $282-288$.